Introduction: describe system & explain question and why it is interesting (2 page)  
Describe model: what is being included/excluded; how do different pieces fit together; derive model/equations(2 page)  
        [somewhere in the Introduction or Description, explain previous relevant models & why yours is different]  
Describe data: what are they? from where do they come? how reliable are they?(1 page)  
Analyze model: explain math/computations; give results(2 page)  
Conclusion: what is the answer to the question, from results? discuss answer; How might model be extended/improved?（1 page）

**Social Media Influencers’ Advertisement in Social Networks and Effect on Cloth Shop in China**

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1. *Introduction*

The social media influencers (abbreviated as SMI) have a rising commercial effect in China [1]. The social media influencers share their lives on social media and have tremendous number of followers who admire their elegant lifestyle and personalities. Served as an “expert” in the specific field, for example fashion, food, cosmetics, these social media influencers would share products they used in the field. Different from the advertisements of celebrities on television, social media influencers have a sense of intimacy to common people and their comments about one product present a feeling of authenticity. Their followers will easily become the potential purchasers of their recommended products. Another advantage is the low cost and spreadability of social media. Because of these advantages of working with social media influencers, cooperating with the social media influencers to facilitate sales becomes a popular choice among companies and retailers in China. In this project, I am going to build a dynamic model to simulate the market of a product and investigate the influence of social media influencer on one product. By building this model, I could answer the questions that is social media influencer working, by how much the social media influencer contributing to the sales and how does different social media influencers perform in my model.

1. *Background*

E-commerce has become a popular trend and is changing people’s lifestyle in China. It breaks the territory restrictions and offers bargains by providing people with various choices of the product. In 2021, the Chinese e-commerce sales reached about 2.64 trillion dollars and boosts up by 12 percentage compared to last year.

The social media influencers, who regularly post their professional suggestions on social media in the form of pictures or videos, own a reputation in one specific field. Attracted by the content of these enthusiastic social media influencers’ posts, people start to follow them. After entrenching their reputation and owning a relatively large number of followers, the social media influencers will have a chance to influence the followers’ decisions and add commercial advertisements on products. These social media influencers not only benefit the large companies, but also attract the small retailers. Before the social media influencers become popular, the small retailers could only sell their products locally and promote interpersonally. The retailers cannot afford other kinds of high-cost advertisements for example television advertisements. For these small-scale retailers, which include the in-season cloth stores, they are able to allow more people to know the existence of the brand and sell clothes online to people who are unable to physically present in the store.

1. *SIR, SIS model and previous researches*

*3.1 SIR model in Epidemiology*

The SIS model is a variation of SIR model, which is derived for the epidemiology to calculate the effect of infectious diseases. The SIR model is a mathematical model by William Ogilvy Kermack and A. G. McKendrick in 1927 [2]. In the context of SIR model in epidemiology, S, I, and R represents the susceptible, infected, and recovered group respectively. The SIR model depicts the dynamic connection between S, I, and R using different change rates from one group moving into another group. In the SIR model, it describes the disease that once the person is infected, he or she will never be infected again in the future. People in susceptible group might be infected by people in infectious group and are able to infect other people in a specific time period. After that time period when they are unable to infect other susceptible people, these people in infectious group will move to the recovered group.

Recovered

Infecting

Susceptible

The SIR model contains two parameters in total. In the model, b represents the average number of people each infected person contacts and infects if the person is able to be infected per unit time. Because per unit time, each infecting person will contact people proportionally based on number of people in each group, namely S, I, and R, there will be bs(t) number of people being contacted and infected by the infecting group. Thus, the rate of change from S group to I group will be bs(t)I(t) per unit time. k is another parameter in the model, and it represents the average proportion of people who will lose the infect ability per unit time. Thus, the rate of change for I to R is kI(t) per unit time. Based on these parameters, the differential equation for the SIR model is the following.

The SIR model makes sense in the case of epidemiology, but it does not make sense in the case of cloth product because it is highly likely for the customers to repurchase the product.

*3.2 SIR model in Market*

The previous research by Riktesh has investigated the blogging effect using SIR model. In specific, the Riktesh’s research calculates the average number of customers who will purchase after reading the post [3]. In Riktesh’s model, it also considers the situation that people who bought the product will infect other people to purchase. However, in the online sales, especially when the product only lasts for a short time, for example 15 days, such “infection” is negligible and hard to distinguish in real data. By removing the “infectious” property in my model, I change my model into SIS instead.

1. *SIS Model in Cloth Market*

In this project, I used the SIS model to simulate the dynamic market of a product in an online shop, which has one social influencer advertising it by having advertisement in their blog. Similar to the SIR model, the SIS model also describes the dynamic change, but only depicts the change within susceptible and infected group. The main difference is that in the setting of the SIS model, people who once had the disease could have the disease again, while they cannot in the SIR model. In the cloth market, it is possible for one person to repurchase the product if he or she likes this product, or the product is a necessity in their lives. Thus, it will be more suitable to use the SIS model to depict the market of a cloth product. Similar to the example in the epidemiology that it uses the status of the person’s infectiveness to distinguish people into different groups, in my model, I used the desire of purchase the product to distinguish people into two groups.

I split the population into two big groups, which are susceptible group and infecting group . People who have potential to purchase the product are considered belong to in susceptible group, and people who have purchased the product and currently have no potential to repurchase the product are in the infecting group. I also divide the people in S(t) into two groups exclusively to better examine the social media influencer’s effects, according to whether or not the person is the fan of the social media influencer. In specific, people who are not fans of social media influencer are considered in , and people who are fans of social media influencer are considered in .

This SIS model captures the change of people’s purchasing motivation and purchase status. If one person never purchased the product before, he or she belongs to the susceptible group. For people in the susceptible group 1 , they have potential to purchase the product, and the rate of those people changing into infecting group is per unit time. For people in susceptible group 2 , they are fans of the social media influencer and rate for their transition is plus the social media influencer’s effect per unit time. Once you purchase the product, you make a transition from susceptible group into infecting group. You will stay in infecting group until you generate a motivation to purchase the product again. And at that time, you will go back to the susceptible group. The rate of infecting people going back to susceptible group is per unit time.

Here is a diagram of my model.

Susceptible1

Infecting

Susceptible2

Based on the rates of change described above, I derive the following differential equations to measure the dynamic change of each susceptible group and infecting group.

\*Where s1, s2 represents the proportion of S1, S2 among total population,

In this model, we will assume that the total number of customers in the market is the same, namely, in the market for each product, the total number of customers remain the same though the number of S and I might change as time changes. We also assume that the people in group will not see the social media influencer’s post and not affected by the social media influencer. Another assumption is that when people in the infecting group regenerating the desire to repurchase the product, they will return to the and group proportionally based on the number of people in these two groups. For instance, if there are 1 person in group, 2 people in group, and 3 people going back from I to S, there will be 1 person return to group and 2 people return to group.

1. *Dataset*

My dataset is from a cloth store in China, and the dataset contains all the history orders of a coat. The dataset records the order time, payment time, and payment account. In total, there are 1073 number of orders for this product and the time duration is approximately 27404 minutes. Here is an overview of the dataset. Graphical user interface

Description automatically generated with low confidence

For this product, almost half of the product is purchased when the product launches on the website, and in the remaining time the sale increases at a decreasing rate. Here is a graph of the sale trend.

Line chart

Description automatically generated with low confidence

Graph 1 Cumulative Sale Trend Per Minute

The orange line represents the original sale trend, and the blue line represents the smoothed sale trend by fitting the data into logarithm function.

The original sale trend has some surges because of manually increasing in stock. There is only limited number of products for the coat. When customers receive the product, and if the product does not fit or they dislike the product, they will ask for a return. After noticing the return, the shop will increase the stock for the product, and thus results in a surge in sale. These surges are unnatural if we consider the stock will automatically whenever there is a return. Thus, I used the smoothened trend, which is represented by the blue line, as the line that I will simulated in the model simulation part.

1. *Model Simulation*

Because all the parameters in the model are arbitrary, I instead use the model simulation to find the parameters. In specific, I tried to find , , , at time 0 and at time 0. I tried to find the best set of parameters that fit the smoothed cumulative purchases of the real data. I used the cumulative number of purchases as the parameters’ key performance indicator because these cumulative number of purchases are my main focus on exanimating the effect of social media influencers.

In this simulation, I used python to solve the differential equation per unit time, and then calculated the cumulative number of purchases. In specific, for per unit time, I timed the rate of change for each susceptible group by the corresponding rate of change. By summing all these values, I could retrieve the simulated cumulative number of purchases. Then, I compared the trend of the simulated cumulative purchase and the smoothed cumulative purchase.

From this simulation, I retrieved the best result as the following.



And the simulated trend is shown below, with an mean absolute error of 328.2.

A picture containing chart

Description automatically generated

From this simulation, it suggests several implications of the market of the product. First, it suggests that the cumulative sales are increasing in a decreasing rate, not only for the non-fan population (S1), but also for the fan population (S2). Second, the fan population will finish almost all of their purchases in the first 200 minutes, while the non-fan population continues to purchase overtime. Third, the social media influencer’s attraction () is 100 times of the attraction other than the social media influencer’s attraction ().

1. *Change of Variables*

In this model, there are two parameters related to the social media influencer, which are the number of people in the S2 and the social media influencer’s attraction rate. Thus, I investigated how does change of these two factors influence the cumulative purchase trend.

* 1. *Ceteris Paribus, but more Attraction to Fans*

If we increase by ten times, which is from 0.02 to 0.2, this indicates the situation that if we are using a social media influencer who have the same amount of fan but be more attractive when persuading the fans to purchase, for example, more attracting words to describe the product, better quality of pictures of product. The graphs of cumulative sales are shown below.

A picture containing text

Description automatically generated A picture containing diagram

Description automatically generated

From the graphs, the total sale only increase by about 50. The starting points for the number of consumptions at time 0 is about 100 higher for the = 0.2, but the overall trends of the two graphs looks the same. However, if we zoom in the cumulative purchases during the first 500 minutes, the number of consumptions for = 0.2 still has a steeper trend and quicker to converge. It makes sense because per unit time, there will be more proportion of people in the S2 group going to the infecting group if is bigger.

A picture containing chart

Description automatically generated

* 1. *Ceteris Paribus, but more Population of Fans*

If we increase by twice, which is from 600 to 1200, this indicates the situation that the social media influencer is having the same post, pictures, and attraction to the fans but with more number of fans.

A picture containing chart

Description automatically generatedA picture containing text

Description automatically generated

The graphs suggest that the total sale will boost by about 600. The overall trends of the two graphs look the similar, but if we zoom in, the fan population with 1200 people have a faster growth rate. This can be explained because given the same fraction of people will go to the infecting group per unit time, if there are more people in the group then more of them will purchase the product per unit time.

Chart, line chart

Description automatically generated

* 1. *Ceteris Paribus, but not Using Social Media Influencer*

If we decrease to zero when we do not use any social media influencer, which is from 600 to 0, the total sales will not increase over time and remains at about 300.

Graphical user interface, application

Description automatically generated

* 1. *Conclusion*

Even though this model is only appliable to the specific product, the coat that released last month, we could still conclude several essential implications in the cloth market from this model and my question.

If we compare the results of 7.1 and 7.2, the graph suggests that if the store wants to have more cumulative purchases overall, it will be more efficient to increase the population of fan, which is S2, compared to . Because from the 7.1, when we increase the by 10 times, the cumulative sale is only increased by about 50. However, if we increase the S2, the cumulative sale will increase at least by the change of S2.

From result 7.3, we could conclude that working with the social media influencer will contribute to the number of purchases of the product. Because if we do not use any social media influencer, as time increases, the total number of purchases will remain around 300 and increase very slowly over time. With the help of social media influencer, it will increase the sale to around 900. So the social media influencer will increase the sales by 600.

1. *Improvements*

In this model, there are some deficiencies. We would only know how the social media influencer will affect the product’s sale when we have such dataset that record the sales per unit time and simulate the parameters. Thus, in the future, I propose the following improvements to make model more predictive:

1. Find the meaningful key performance indicator to estimate α, β, and μ. Because the parameters α, β, and μ are arbitrary and not connected to the real life. This can be solved if in the future we can estimate parameters for example μ using the repurchasing rate.
2. Try to approximate the population in S1, S2 before the model. For example, if we can try to use previous dataset to find the average S1 and S2 if we are using the same cloth shop and social media influencer.

*References*

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